

## Research Article

# Monotone Generalized Nonlinear Contractions in Partially Ordered Metric Spaces

Ljubomir Ćirić,<sup>1</sup> Nenad Cakić,<sup>2</sup> Miloje Rajović,<sup>3</sup> and Jeong Sheok Ume<sup>4</sup>

<sup>1</sup> Faculty of Mechanical Engineering, University of Belgrade, Kraljice Marije 16,  
11 000 Belgrade, Serbia

<sup>2</sup> Faculty of Electrical Engineering, University of Belgrade, Boulevard Kralja Aleksandra 73,  
11 000 Belgrade, Serbia

<sup>3</sup> Faculty of Mechanical Engineering, University of Kragujevac, Dositejeva 19,  
36 000 Kraljevo, Serbia

<sup>4</sup> Department of Applied Mathematics, Changwon National University,  
Changwon 641-773, South Korea

Correspondence should be addressed to Ljubomir Ćirić, lciric@rcub.bg.ac.yu

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A concept of  $g$ -monotone mapping is introduced, and some fixed and common fixed point theorems for  $g$ -non-decreasing generalized nonlinear contractions in partially ordered complete metric spaces are proved. Presented theorems are generalizations of very recent fixed point theorems due to Agarwal et al. (2008).

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## 1. Introduction

The Banach fixed point theorem for contraction mappings has been extended in many directions (cf. [1–28]). Very recently Agarwal et al. [1] presented some new results for generalized nonlinear contractions in partially ordered metric spaces. The main idea in [1, 20, 26] involve combining the ideas of iterative technique in the contraction mapping principle with those in the monotone technique.

Recall that if  $(X, \leq)$  is a partially ordered set and  $F : X \rightarrow X$  is such that for  $x, y \in X$ ,  $x \leq y$  implies  $F(x) \leq F(y)$ , then a mapping  $F$  is said to be non-decreasing. The main result of Agarwal et al. in [1] is the following fixed point theorem.

**Theorem 1.1** (see [1, Theorem 2.2]). *Let  $(X, \leq)$  be a partially ordered set and suppose there is a metric  $d$  on  $X$  such that  $(X, d)$  is a complete metric space. Assume there is a non-decreasing function  $\psi : [0, +\infty) \rightarrow [0, +\infty)$  with  $\lim_{n \rightarrow \infty} \psi^n(t) = 0$  for each  $t > 0$  and also suppose  $F$  is a non-decreasing*





















